Non-Strange Baryon Resonances in the Isgur-Karl Model Including the $N = 3$ Oscillator Shell

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Z. Naturforsch. **49a**, 733–745 (1994); received December 8, 1993

The nonrelativistic constituent quark model for excited baryon states in the version proposed by Isgur and Karl is extended toward higher energies by including all states containing up to three oscillator excitation quanta. At high energies baryon resonances with spins up to 9/2 become accessible. At all energies and in particular for lower spins strong configuration mixing is obtained. Model predictions are compared to the empirical resonance energies derived in the Karlsruhe-Helsinki pion nucleon phase shift analysis. A search is made for parameters sets that apply simultaneously to the positive parity and the negative parity states. Numerical results are presented and discussed in relation to experimental data and to other theoretical works.

PACS numbers: 12.40.Aa, 12.40.Qq, 14.20.Gk.

1. Introduction

The nonrelativistic constituent quark model as formulated by Isgur and Karl [1] has been highly successful, in a semi-quantitative way, in describing excitation energies, spins and parities of observed baryon ground states and resonances [1]. The model is based on a harmonic oscillator confining potential with anharmonic corrections and a hyperfine interaction inspired by the 1-gluon exchange graph of QCD. Spin-orbit forces are assumed to be negligible [6]. The wave functions are, contrary to bag models, fully translation invariant and can be roughly classified according to an oscillator shell model. Due to hyperfine and anharmonic perturbing interactions, the eigenstates of the hamiltonian are not pure shell model states, but exhibit configuration mixing between the $N = 0$ and $N = 2$ oscillator shells in the case of positive parity states, and between the $N = 1$ and $N = 3$ oscillator shells in the case of negative parity states. For positive parity states configuration mixing has been shown to lead to interesting physical consequences explaining the electric charge form factor of the neutron [7] and the violations of certain $SU(6)$ selection rules [8]. For negative parity states only very few authors have attempted to go beyond the $N = 1$ shell, because of the greatly increased computational effort that is required, when the $N = 3$ shell is also included.

This paper reports work in which the additional complexity arising from the inclusion of the $N = 3$ shell was fully taken into account. Previous studies by Bowler et al. [9] have analyzed by group theoretical methods the splitting of the pure $N = 3$ oscillator level under the influence of anharmonic perturbations, however without considering the additional splitting due to hyperfine interactions. Their work has been continued by Corvi [10] and by Forsyth and Cutkosky [11], who have analyzed certain subsets of the $N = 3$ energy levels with inclusion of the hyperfine interaction. The last named authors [12] have also performed a full analysis of the $N = 3$ oscillator shell similar to the work reported in this paper, but they have used, as will be discussed in more detail below, somewhat different assumptions on their model, and they have based their numerical calculations on the Carnegie-Mellon-LBL (CMU-LBL) [13] pion-nucleon phase shift analysis as opposed to the Karlsruhe-Helsinki (KH) [14, 15] analysis, which was used in our work.

2. The Baryon Model of Isgur and Karl

The aim of our calculations has been (i) to derive the predictions of the Isgur-Karl model in its "orthodox" form with the $N = 3$ oscillator shell included, and (ii)

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[1] A comprehensive survey of the constituent quark model and baryon resonances has been given by Hey and Kelly [2]. For a review of general aspects of the non-relativistic 3-body problem with special emphasis on the physics of hadrons see Richard [3]. The theoretical foundations of the non-relativistic constituent quark model are discussed by Lucha, Schöberl, and Gromes [4]. For an introduction to and a recent status report on the Isgur-Karl model see Capstick [5].

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to use these results as a reference point from which various modifications of the model can be judged as to whether or not they improve the agreement with experimental data.

The hamiltonian of the Isgur-Karl model [1] consists of three pieces,

$$H = H_0 + U + H_{\mathrm{hyp}}, \tag{1}$$

describing quark confinement, anharmonic perturbations, and hyperfine interactions, respectively. The requirement of translation invariance suggests the use of Jacobi coordinates

$$\mathbf{x}_\varrho = \frac{1}{\sqrt{2}}(\mathbf{x}_1 - \mathbf{x}_2), \quad \mathbf{x}_\lambda = \frac{1}{\sqrt{6}}(\mathbf{x}_1 - \mathbf{x}_2 - 2\mathbf{x}_3). \tag{2}$$

The confinement hamiltonian H_0 describes the internal motion of three quarks of mass m, which are bound by the confining harmonic potential

$$\begin{aligned} V_0 &= \frac{m\omega^2}{6} \cdot [(\mathbf{x}_1 - \mathbf{x}_2)^2 + (\mathbf{x}_2 - \mathbf{x}_3)^2 + (\mathbf{x}_3 - \mathbf{x}_1)^2] \\ &= \frac{m\omega^2}{2} \cdot (\mathbf{x}_\varrho^2 + \mathbf{x}_\lambda^2). \end{aligned} \tag{3}$$

The hamiltonian H_0 is given by

$$H_0 = 3m + \frac{1}{2m}(\mathbf{p}_\varrho^2 + \mathbf{p}_\lambda^2) + \frac{1}{2} m\omega^2 (\mathbf{x}_\varrho^2 + \mathbf{x}_\lambda^2). \tag{4}$$

The use of harmonic potentials is an overidealization, and the second term U in (1), describing spin-independent anharmonic 2-body forces and possibly 3-body forces, is introduced to correct this circumstance. This piece will be further discussed below. The third term (with $\mathbf{x}_{ij} = \mathbf{x}_i - \mathbf{x}_j$, $r_{ij} = |\mathbf{x}_{ij}|$, and $i, j = 1, 2, 3$)

$$\begin{aligned} H_{\mathrm{hyp}} = \sum_{i<j} \frac{2\alpha_s}{3m^2} \Bigg[& \frac{8\pi}{3} (\mathbf{s}_i \cdot \mathbf{s}_j)\, \delta^3(\mathbf{x}_{ij}) \\ & + \frac{1}{r_{ij}^3} \left(\frac{3(\mathbf{s}_i \cdot \mathbf{x}_{ij})(\mathbf{s}_j \cdot \mathbf{x}_{ij})}{r_{ij}^2} - (\mathbf{s}_i \cdot \mathbf{s}_j) \right) \Bigg] \end{aligned} \tag{5}$$

describes the spin-dependent short range quark-quark interaction that is suggested by the 1-gluon exchange graph of QCD. For the purpose of the Isgur-Karl model the strength of the hyperfine interaction, in other words the effective QCD coupling constant α_s, is considered to be an adjustable parameter.

The prescription by Isgur and Karl requires the hamiltonian $H = H_0 + U + H_{\mathrm{hyp}}$ to be investigated in the finite dimensional truncated Hilbert space $\mathscr{H}^{(N)}$ that is spanned by the eigenfunctions of H_0 with up to N oscillator excitation quanta. In $\mathscr{H}^{(N)}$, with the eigenfunctions of H_0 chosen as a basis, the operators U and H_{hyp} are represented by (in general) nondiagonal hermitean matrices $U^{(N)}$ and $H_{\mathrm{hyp}}^{(N)}$. The total hamiltonian H is represented in the truncated Hilbert space $\mathscr{H}^{(N)}$ by the $N \times N$-matrix $H^{(N)}$, which must now be numerically diagonalized to obtain its eigenvalues. These are then compared to the masses of experimentally known baryon resonances. The adjustable parameters of the model must be determined so that in some sense a "best fit" is obtained. Available parameters are m, ω, α_s, and several parameters derived from the operator U.

3. The Spin-Independent Anharmonic Perturbation U

All spin-independent forces acting in the three-quark system are supposed to be derivable from a local potential [16]

$$\begin{aligned} & V(\mathbf{x}_\varrho^2, \mathbf{x}_\lambda^2, (\mathbf{x}_\varrho \cdot \mathbf{x}_\lambda)) \\ & = \tfrac{1}{2} m\omega^2 (\mathbf{x}_\varrho^2 + \mathbf{x}_\lambda^2) + U(\mathbf{x}_\varrho^2, \mathbf{x}_\lambda^2, (\mathbf{x}_\varrho \cdot \mathbf{x}_\lambda)). \end{aligned} \tag{6}$$

The harmonic term has been separated from U for the purely technical reason that combined with the kinetic energy terms it yields the exactly soluble confining hamiltonian H_0. The oscillator constant ω should be chosen so that in the region of $(\mathbf{x}_\varrho, \mathbf{x}_\lambda)$-space, where the wavefunctions of our truncated Hilbert space differ appreciably from zero, the remainder term $U(\mathbf{x}_\varrho^2, \mathbf{x}_\lambda^2, (\mathbf{x}_\varrho \cdot \mathbf{x}_\lambda))$ is in some sense minimized. There are good reasons to believe that U can be well approximated by a sum of 2-body potentials U_2 [17], but U may also contain a term describing a genuine 3-body potential U_3 [18].

$$\begin{aligned} & U(\mathbf{x}_\varrho^2, \mathbf{x}_\lambda^2, (\mathbf{x}_\varrho \cdot \mathbf{x}_\lambda)) \\ & = U_2(r_{12}) + U_2(r_{13}) + U_2(r_{23}) + U_3(\mathbf{x}_\varrho^2, \mathbf{x}_\lambda^2, (\mathbf{x}_\varrho \cdot \mathbf{x}_\lambda)). \end{aligned} \tag{7}$$

When the 3-body potential U_3 is neglected, the matrix elements of the truncated operator $U^{(N)}$ can be parametrized in terms of $N+1$ constants

$$a_n = 12\pi \left(\frac{\alpha}{2\pi}\right)^{n+\frac{1}{2}} \int_0^\infty \mathrm{d}x\, U_2(x)\, x^{2n} \exp(-\tfrac{1}{2}\alpha x^2), \qquad (n = 1, 2, \ldots, N+1) \tag{8}$$

with $\alpha = \sqrt{m\omega}$. The first four of these parameters are usually denoted as $a = a_1$, $b = a_2$, $c = a_3$, $d = a_4$. When up to $N = 3$ oscillator quanta are taken into account, the following combinations are very convenient:

$$\begin{aligned} E_0 &= 3m + 3\omega + a, \\ \Omega &= \omega - \tfrac{1}{2}a + \tfrac{1}{3}b, \\ \Delta_1 &= -\tfrac{5}{4}a + \tfrac{5}{3}b - \tfrac{1}{3}c, \\ \Delta_2 &= +\tfrac{1}{2}b - \tfrac{2}{5}c + \tfrac{2}{35}d. \end{aligned} \tag{9}$$

In the case that one restricts attention to one oscillator shell at a time and neglects matrix elements connecting different harmonic oscillator shells, the eigenvalues of the operator $H_0^{(N)} + U^{(N)}$ can be explicitly stated in terms of E_0, Ω, Δ_1, Δ_2. For the detailed results we refer to the paper by Bowler et al. [9].

The calculation reported in this paper, however, does consider several oscillator shells simultaneously, and therefore the neglect of these matrix elements is here not justified. Because of parity conservation, only oscillator shells differing by an even number of excitation quanta can mix, i.e. mixing occurs either between the $N=0$ and $N=2$ shells or between the $N=1$ and $N=3$ shells. Taking into account the implications of rotational and permutational symmetry, one concludes that possible mixing of the $N=0$ and $N=2$ shells must be effected by a term proportional to

$$\mathbf{a}_\varrho \cdot \mathbf{a}_\varrho + \mathbf{a}_\lambda \cdot \mathbf{a}_\lambda + \mathbf{a}_\varrho^\dagger \cdot \mathbf{a}_\varrho^\dagger + \mathbf{a}_\lambda^\dagger \cdot \mathbf{a}_\lambda^\dagger, \tag{10}$$

where $\mathbf{a}$ and $\mathbf{a}^\dagger$ are the annihilation and creation operators of the ϱ and λ oscillators that make up the hamiltonian H_0. Since the potential U is assumed to be local, i.e. independent of the momenta $\mathbf{p}_\varrho$ and $\mathbf{p}_\lambda$, the expression in (10) can arise only from a term const $\cdot (\mathbf{x}_\varrho^2 + \mathbf{x}_\lambda^2)$, where we are using $\mathbf{x}_\varrho = (2m\omega)^{-1/2}(\mathbf{a}_\varrho + \mathbf{a}_\varrho^\dagger)$ and $\mathbf{x}_\lambda = (2m\omega)^{-1/2}(\mathbf{a}_\lambda + \mathbf{a}_\lambda^\dagger)$. A term const $\cdot (\mathbf{x}_\varrho^2 + \mathbf{x}_\lambda^2)$ contained in U would be a harmonic contribution contained in U, which can be transferred from U to H_0 by an appropriate adjustment of the oscillator frequency ω. For this reason we can assume with no loss of generality that with the optimum choice of ω the potential U induces no mixing between the $N=0$ and $N=2$ oscillator shells. Because the mixing matrix elements are proportional to $\Omega - \omega$, the non-occurrence of mixing between the $N=0$ and $N=2$ oscillator shells implies $\Omega = \omega$ (cf. the numerical results discussed in Section 5).

The situation is quite different when we look at the possible mixing between the $N=1$ and $N=3$ oscillator shells. The $N=3$ shell contains two multiplets with mixed permutational symmetry, negative parity, and orbital angular momentum $L=1$, usually denoted as $[70,1^-]_{N=3}^{(i)}$ $(i=a,b)$. These have the same symmetry properties as the states $[70,1^-]_{N=1}$ and differ from them only by radial excitations. Consequently, the matrix elements $\langle [70,1^-]_{N=1} | U | [70,1^-]_{N=3}^{(i)} \rangle$ turn out to be nonvanishing. Explicit computation shows that under the assumption $\Omega = \omega$ these matrix elements are proportional to the parameter Δ_1. In numerical calculations Δ_1 is a comparatively large quantity. Strong mixing of the $[70,1^-]$ states contained in the $N=1$ and $N=3$ oscillator shells due to the anharmonic potential U is therefore absolutely unavoidable and can not be eliminated by a reshuffling of terms as in the positive parity case. The mixing tends to lower the $N=1$ states and to raise to $N=3$ states.

As will be discussed in more detail below, the parameter Δ_2 is not well determined by the fitting procedures to the experimental resonance energies. It has been pointed out by Forsyth and Cutkosky [12] that a confining potential as the one that one expects in the framework of QCD (Coulomb plus linear term) would lead to a small negative value for Δ_2.

The possible influence of the 3-body potential U_3 is largely unknown. When U is a sum of 2-body potentials, $U = U_2(r_{12}) + U_2(r_{23}) + U_2(r_{13})$, the splitting of the $N=2$ oscillator shell under the influence of U is completely determined by the parameter Δ_1, but when a 3-body term U_3 is added, this is no longer true for the $[56,0^+]_{N=2}$ state, whose splitting now depends on further parameters [16, 19]. Similar effects of the 3-body forces must be expected for negative parity states, and presumably some of the $N=3$ states will not follow the splitting pattern predicted under the assumption that U is built with 2-body forces only. For results obtained with hyperspherical potentials see Richard and Taxil [20].

4. Method of Calculation

For comparison with experimental data a computational scheme has to be set up, by which for a given set of parameters a theoretical prediction is obtained, that consists of the eigenvalues and the corresponding eigenvectors of the matrix $H^{(N)}$, which is the total hamiltonian restricted to the truncated Hilbert space $\mathscr{H}^{(N)}$. These eigenvalues are then compared to a set of baryon resonance parameters obtained from phase shift analyses and inelastic reactions. Systematic variations of the available parameters lead to a "best fit" of the experimental data.

To set up the computational scheme we have first constructed all wave functions of non-strange states with up to $N=3$ oscillator excitations. The configuration space parts of the wave functions were obtained in agreement with the creation operator polynomials derived by Bowler et al. [9], these were then combined with non-strange $SU(6)$ spin-isospin wave functions into wave functions symmetric under permutations of the three quarks. In addition one also has the totally antisymmetric color factor that takes care of the Pauli principle. With these wave functions the matrix elements of the anharmonic perturbation U and of the hyperfine interaction H_{hyp} were computed and tabulated, using standard methods described by Isgur and Karl [1]. Collecting these results one obtains the matrix $H^{(N=3)}$, which is then diagonalized numerically. The eigenvalues correspond to the energies of baryon states predicted by the model, the eigenvectors provide information about the configuration mixing in these states.

For the optimization of the parameters a number of experimentally well known resonances (mostly with 4*-status) were selected and compared to the model predictions. The function

$$F=\sum_i \frac{(E_i^{\exp}-E_i^{\text{theor}})^2}{(\Gamma_i^{\exp})^2}, \tag{11}$$

where the sum extends over the selected resonances, was minimized by a computer routine. The nucleon $N(939)$ and the $\Delta(1232)$ were given special treatment. For these very large (but arbitrary) weight factors were introduced to ensure that their masses would come out right. All other resonances have been weighted by the inverse squares of the experimental widths rather than the socalled "errors" of the experimental resonance energies given in the literature. These errors have some meaning within the systematic procedures of one particular data analysis, but they generally underestimate the uncertainty of the experimental resonance parameters. This becomes evident when one compares the works of different groups. In particular, for broad inelastic resonances the results depend on the resonance definition that has been adopted and on the method of analysis [2]. In our context the optimization procedure based on (11) always led to reasonable and numerically stable results, whereas corresponding calculations that used the "errors" of the resonance energies led to results strongly dependent on small changes of the "errors".

5. Numerical Aspects

The Isgur-Karl model with up to $N=3$ oscillator excitations predicts 14 positive parity and 35 negative parity nucleon states, 9 positive parity and 17 negative parity Δ-states, altogether 75 nonstrange baryon states with spins ranging up to 9/2. On the experimental side the Particle Data Group tables [21] list 19 nucleon resonances and 18 Δ-resonances with corresponding quantum numbers, among these there are 10 nucleon and 6 Δ-states rated as 4*-resonances. Our theoretical model provides us with 6 parameters, namely the harmonic oscillator constant ω contained in H_0, the constants E_0, Ω, Δ_1, Δ_2, describing the anharmonic perturbation U, and a constant δ determined by the strength of the hyperfine interaction H_{hyp}. In accordance with the work of Isgur and Karl [1] this parameter is defined as

$$\delta=\frac{4\alpha_s\,\omega^{3/2}}{3\sqrt{2\pi m}}, \tag{12}$$

where m is the constituent quark mass and α_s the effective QCD coupling constant.

By comparison with the experimental data it turns out that all of these parameters except Δ_2 are determined by the data within rather narrow limits. Below we describe four out of a larger number of fits. Except where noted otherwise the input data have been the resonance energies and widths obtained by the Karlsruhe-Helsinki (KH) phase shift analysis [14, 21]. The optimal parameters determined by these fits are collected in Table 1, the resonance energies predicted by the Isgur-Karl model are listed in Table 2 together with "experimental" values derived from phase shift analyses.

Table 1. Model parameters determined by the four fits described in the text.

Fit	E_0	δ	ω	Ω	Δ_1	Δ_2	$\delta E_{70,1}$	$\delta E_{56,3}$
a	1134	260	478	480	505	−22		
b	1140	260	473	473	573	140		
c	1140	260	473	473	573	631		
d	1140	260	473	473	574	−22	274	−134

[2] For a detailed discussion of the existing problems and the discrepancies found in the literature we refer to the handbook article by Höhler [15, p. 202].

Table 2. Resonance energies predicted by our four fits to the Isgur-Karl model. Comparison is made to the resonance energies deduced from the experimental data by the Karlsruhe-Helsinki ("Höhler") and the Carnegie-Mellon-Berkeley ("Cutkosky") phase shift analyses, and to some additional data as indicated. All data have been quoted according to the 1990 Review of Particle Properties [21].

Table 2.1. Nucleon states with positive parity.

State	Theoretical results				Experimental data				
	Fit (*a*)	Fit (*b*)	Fit (*c*)	Fit (*d*)	Höhler	Cutkosky	Others	Status	Width
P_{11}	939	939	939	939	939	939		****	Stable
	1456	1384	1384	1380	1410	1440		****	$\Gamma \approx 200$
	1774	1734	1734	1735	1723	1700		***	$\Gamma \approx 110$
	1962	1940	1940	1941					
	2121	2110	2110	2111	2050	2125		*	$\Gamma \approx 230$
P_{13}	1775	1742	1742	1743	1710	1700		****	$\Gamma \approx 200$
	1906	1867	1867	1867					
	2002	1977	1976	1977					
	2029	2007	2007	2008					
	2117	2106	2106	2107					
F_{15}	1780	1747	1747	1748	1684	1680		****	$\Gamma \approx 125$
	2004	1980	1979	1980	1882		1970 [a]	**	$\Gamma \approx 120$
	2070	2048	2048	2049					
F_{17}	2009	1987	1987	1987	2005	1970		**	$\Gamma \approx 350$

[a] Seen by Langbein and Wagner in $\pi N \to \Sigma K$ [25].

Fit (a): For a first orientation we have selected 6 well known resonances to fix our 6 parameters, namely $N\frac{1}{2}^+$(939), $N\frac{1}{2}^+$(1440), $\Delta\frac{3}{2}^+$(1232), $\Delta\frac{7}{2}^+$(1950), $N\frac{5}{2}^-$(1675), and $N\frac{9}{2}^-$(2250). The first 5 resonances fix the parameters E_0, δ, ω, Ω, and Δ_1, while the last one determines Δ_2. Using the resonance energies obtained by the Karlsruhe-Helsinki (KH) phase shift analysis [14, 21] we found $E_0 = 1134$ MeV, $\delta = 260$ MeV, $\omega = 478$ MeV, $\Omega = 480$ MeV, $\Delta_1 = 505$ MeV, and $\Delta_2 = -22$ MeV. These values are in good agreement with the original analyses by Isgur and Karl [1]. It is interesting to note the close proximity between ω and Ω, which indicates the absence of mixing that would come from the anharmonic perturbation U. The configuration mixing required for the simultaneous fit of the nucleon $N\frac{1}{2}^+$(939) and the Roper resonance $N\frac{1}{2}^+$(1440) must therefore be solely due to the hyperfine interaction [1, 8]. The coincidence $\omega \approx \Omega$ persisted through all subsequent calculations.

Fit (b): In a second round of calculations we tested how sensitively the model parameters depend on the particular choice of input data. Generally speaking, E_0, δ, and $\omega \approx \Omega$ showed little variation in all fits that included the nucleon, the Δ(1232), and the Roper resonance $N\frac{1}{2}^+$(1440). The anharmonic parameter Δ_1 naturally exhibits some dependence on our choice of higher energy states, while Δ_2 is strongly dependent on the choice of input data and on the method of calculation. As an example we quote the results of a fit that used all positive parity 4*-resonances (as determined by the Karlsruhe-Helsinki phase shift analysis) for the optimization of the model parameters E_0, δ, ω, Ω, and Δ_1, and the $N\frac{9}{2}^-$(2250) 4*-resonance for the determination of Δ_2. The parameters were obtained as follows: $E_0 = 1140$ MeV, $\delta = 260$ MeV, $\omega = 473$ MeV, $\Omega = 473$ MeV, $\Delta_1 = 573$ MeV, and $\Delta_2 = 140$ MeV.

Fit (c): When in addition to the $N\frac{9}{2}^-$(2250) resonance all remaining negative parity 4*-resonances are included in the input data, the first five parameters remain unchanged, but Δ_2 increases dramatically to $\Delta_2 = 620$ MeV.

Comparing the model prediction to the experimental data, we note through all these fits two major discrepancies:

(i) The lowest negative parity states that are thought to contain a P-state assigned to the $N = 1$ oscillator shell as their predominant component [$N\frac{1}{2}^-$(1535), $N\frac{1}{2}^-$(1650), $N\frac{3}{2}^-$(1520), $N\frac{3}{2}^-$(1700), $N\frac{5}{2}^-$(1675), $\Delta\frac{1}{2}^-$(1620), $\Delta\frac{3}{2}^-$(1700)] all come out too low by amounts between 120 and 170 MeV. Earlier calculations that were restricted to odd parity states coming from the $N = 1$ oscillator shell did not encounter this difficulty since they adjusted E_0 to fit the average

Table 2.2. Nucleon states with negative parity.

State	Theoretical results				Experimental data				
	Fit (*a*)	Fit (*b*)	Fit (*c*)	Fit (*d*)	Höhler	Cutkosky	Others	Status	Width
S_{11}	1404	1392	1423	1506	1526	1550		****	$\Gamma \approx 150$
	1497	1494	1548	1641	1670	1650		****	$\Gamma \approx 150$
	1884	1821	1844	1741	1880			*	$\Gamma \approx 95$
	2050	2109	2148	2051					
	2215	2165	2173	2163		2180		*	$\Gamma \approx 350$
	2252	2339	2368	2209					
	2330	2383	3059	2309					
	2399	2488	3149	2373					
	2418	2577	3343	2435					
D_{13}	1410	1397	1432	1508	1519	1525		****	$\Gamma \approx 125$
	1556	1558	1630	1690	1731	1675		***	$\Gamma \approx 100$
	1883	1818	1839	1735		1880	1910[a]	**	$\Gamma \approx 180$
	2117	2130	2159	2094	2081	2060		**	$\Gamma \approx 300$
	2226	2230	2265	2165					
	2304	2278	2292	2270					
	2323	2302	2375	2285					
	2332	2375	2504	2303					
	2359	2433	3066	2315					
	2418	2496	3209	2378					
	2441	2642	3424	2502					
D_{15}	1525	1525	1586	1668	1679	1675		****	$\Gamma \approx 155$
	2107	2098	2154	2046					
	2217	2176	2199	2164	2228	2180	1910[a]	**	$\Gamma \approx 350$
	2281	2265	2291	2232					
	2309	2285	2340	2258					
	2319	2323	2450	2281					
	2358	2353	2555	2334					
	2384	2409	2581	2375					
	2419	2545	3335	2398					
G_{17}	2111	2100	2158	2050	2140	2200	2140[b]	****	$\Gamma \approx 390$
	2271	2258	2336	2209					
	2292	2293	2447	2233					
	2355	2335	2534	2313					
	2393	2383	2547	2350					
G_{19}	2249	2268	2513	2187	2268	2250	2200[b]	****	$\Gamma \approx 300$

[a] Bell et al. [23] and Saxon et al. [24] find evidence for a resonance at 1900 to 1920 in the inelastic channel $\pi^- p \rightarrow \Lambda K^0$.
[b] Hendry [26].

Table 2.3. Delta states with positive parity.

State	Theoretical results				Experimental data				
	Fit (*a*)	Fit (*b*)	Fit (*c*)	Fit (*d*)	Höhler	Cutkosky	Others	Status	Width
P_{31}	1906	1866	1866	1866	1888	1910	1715[a]	****	$\Gamma \approx 220$
	1948	1910	1909	1910					
P_{33}	1232	1232	1231	1231	1232	1232		****	$\Gamma \approx 115$
	1765	1695	1695	1692	1522	1600		**	$\Gamma \approx 260$
	1937	1932	1932	1933	1868	1920	1955[b]	***	$\Gamma \approx 250$
	2064	2010	2010	2011					
F_{35}	1974	1938	1938	1939	1905	1910		****	$\Gamma \approx 300$
	2027	2005	2005	2006		2200	2000[c]	**	$\Gamma \approx 400$[d]
F_{37}	1949	1914	1914	1914	1913	1950		****	$\Gamma \approx 240$
		see footnote[e]			2425	2350		*	$\Gamma \approx 300$

[a] Chew reports four resonances at 1715, 1778, 1960, and 2121 MeV respectively [27].
[b] Chew reports two resonances at 1955 and 2065 MeV respectively [27].
[c] Manley [28].
[d] Cutkosky [13].
[e] This resonance would require the inclusion of $N=4$ oscillator states and is not contained in our theoretical calculation.

Table 2.4. Delta states with negative parity.

State	Theoretical results				Experimental data				
	Fit (*a*)	Fit (*b*)	Fit (*c*)	Fit (*d*)	Höhler	Cutkosky	Others	Status	Width
S_{31}	1533	1533	1597	1673	1610	1620		****	$\Gamma\approx 140$
	2143	2055	2056	1921	1908	1890	1919 [a]	***	$\Gamma\approx 150$
	2280	2224	2225	2231		2150	2047 [b]	*	$\Gamma\approx 200$
	2352	2558	3349	2402					
D_{33}	1530	1530	1593	1671	1680	1710		****	$\Gamma\approx 250$
	2117	2027	2027	1895		1940	2058 [a]	*	$\Gamma\approx 200$
	2233	2187	2192	2183					
	2314	2258	2258	2258					
	2360	2335	2335	2334					
	2379	2560	3349	2414					
D_{35}	2133	2043	2043	1910	1901	1940	1910 [a]	***	$\Gamma\approx 250$
	2256	2248	2252	2193					
	2311	2281	2332	2255	2305	2400		*	$\Gamma\approx 350$
	2376	2332	2522	2333					
G_{37}	2257	2264	2269	2195	2215	2200	2280 [c]	*	$\Gamma\approx 400$
	2327	2281	2522	2271					
G_{39}	2292	2236	2236	2237	2468	2300	2200 [c]	**	$\Gamma\approx 450$

[a] Chew [27].
[b] Chew sees two resonances at 2047 and 2203 MeV respectively [27]. [c] Hendry [26].

energy of the states of the $[70,1^-]_{N=1}$ multiplet. In the present calculation this discrepancy can be traced to the fact that the configuration mixing due to the anharmonic perturbation, which is present in the negative parity sector, tends to depress the low lying states and to raise the high lying states. Even with $\omega=\Omega$ the mixing matrix elements are nonvanishing, because they depend on the large parameters Δ_1 and on Δ_2. In fact, the large variation of Δ_2 that we obtained when we included all negative parity 4*-resonances in our input data, can be seen as an attempt of the computer to improve the fit of the low lying P-wave states by reducing the amount of configuration mixing.

(ii) States that derive predominantly from the $SU(6)$ multiplet $[56,1^-]_{N=3}$, notably the $\Delta\frac{1}{2}^-(1900)$ and the $\Delta\frac{5}{2}^-(1930)$, both classified as 3*-resonances, are predicted at energies too high by about 130 MeV. This discrepancy has already been noticed by previous investigators [10, 12], it has as yet not found a satisfactory explanation. In agreement with the previous analyses we find the above mentioned Δ-states to be rather pure $[56,1^-]_{N=3}$ states, so that the conclusion that our $[56,1^-]_{N=3}$ multiplet is too high by 140 MeV appears unavoidable. The results of Richard and Taxil [20] seem to indicate that the $[56,1^-]_{N=3}$ multiplet is in fact strongly dependent on the details of the anharmonic perturbation.

These two observations have led us to yet another attempt to fit the data, reported here as fit (d).

Fit (d): In a final round of calculations we have introduced two additional free parameters by allowing the computer to choose the optimal values for the energies of the $SU(6)$ multiplets $[70,1^-]_{N=1}$ and $[56,1^-]_{N=3}$. As expected, with two additional parameters the fit to the experimental data was considerably improved, however, the procedure lacks a deeper theoretical justification, and also subsequent studies on the pion decay of baryon resonances [22] showed that the baryon wave functions so obtained have some undesirable features.

6. Comparison with the Experimental Data

This section presents a detailed comparison of our model calculations (see Table 2 and Fig. 1) to experimental data as contained in the 1990 Review of Particle Properties (RPP) [21]. Special states will be commented upon, discrepancies will be pointed out, and attention will be drawn to interesting pecularities. The material is organized according to the partial waves in pion-nucleon scattering in which a particular resonance state would be expected to be observed.

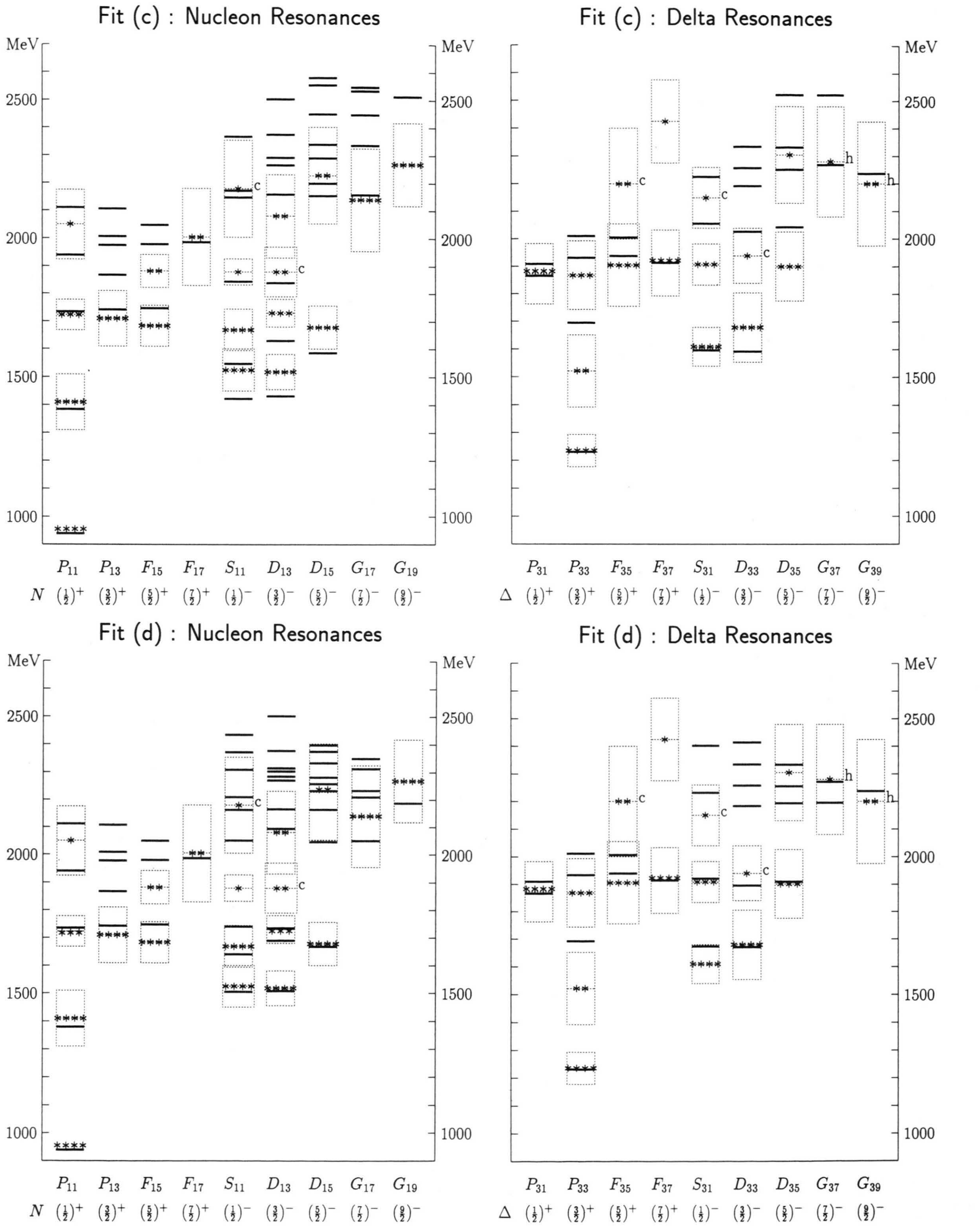

Fig. 1. The theoretical predictions (heavy horizontal bars) of our fits (*c*) and (*d*) are compared to empirical resonance data (unmarked data: Karlsruhe-Helsinki analysis; data marked "c": resonances seen only in the Carnegie-Mellon-Berkeley analysis; data marked "h": from the high partial wave analysis of Hendry [23]). Each resonance is represented by a rectangular box, centered (horizontal dashed line) at the empirical resonance energy and with a vertical size corresponding to the empirical full width of the resonance. The asterisks indicate the quality of the data. The vertical scales measure energy in MeV.

Nucleon States with Positive Parity

$\boldsymbol{P_{11}}$: Our calculation was set up so that the nucleon mass had to be reproduced exactly. As in the work of previous authors the nucleon is found to be mainly a $[56,0^+]_{N=0}$ state with sizable admixtures from the $[56,0^+]_{N=2}$ and $[70,0^+]_{N=2}$ multiplets [3]. The Roper 4*-resonance $N(1440)$ is mainly a $[56,0^+]_{N=2}$ state with appreciable admixture of $[56,0^+]_{N=0}$, its mass comes out 30–40 MeV too low. The 3*-resonance $N(1710)$ is reproduced in a satisfactory manner. The highest known resonance $N(2100)$ (1*-status) is broad and not well determined, it can be accommodated in the model.

$\boldsymbol{P_{13}}$: The 1990 RPP lists a 1*-resonance $N(1540)$, the existence of which is considered as doubtful. The Isgur-Karl model provides no place for such a state. The lowest state predicted by the model has a mass of about 1742 MeV, in fair agreement with the 4*-resonance $N(1720)$ seen in this partial wave [4].

$\boldsymbol{F_{15}}$: In this partial wave we find the well known 4*-resonance $N(1680)$. Its predicted mass is too high by about 60 MeV. Theoretically this state derives, like the $N(1720)$ resonance in the P_{13} wave, from the $[56,2^+]_{N=2}$ and $[70,2^+]_{N=2}$ multiplets [4], it has a similar mass and differs from the $N(1720)$ only by its angular momentum coupling. It is difficult to see how a greater mass splitting could be generated. At higher energy the 1990 RPP lists a 2*-resonance $N(2000)$. The reported mass values range from 1882 MeV (KH) to 2175 MeV, it is not seen in the CMU-LBL phase shift analysis. The Isgur-Karl model could accommodate two states in this energy range.

$\boldsymbol{F_{17}}$: The only known resonance in this partial wave is $N(1990)$ with a 2*-rating. It fits well into the model.

Nucleon States with Negative Parity

$\boldsymbol{S_{11}}$: The 1990 RPP contains two well established 4*-resonances $N(1535)$ and $N(1650)$. For both of them the wave function is predominantly contained in the $[70,1^-]_{N=1}$ multiplet, but with significant admixtures from the $[56,1^-]_{N=3}$ multiplet and the high lying $[70,1^-]^b_{N=3}$ multiplet [5]. For both states the theoretical values are too low by roughly 100 MeV. This feature has persisted to varying degrees in all fits that we have attempted, it seems to arise from the configuration mixing between the $N=1$ and $N=3$ oscillator shells that is caused by the nondiagonal matrix elements of the anharmonic potential U. A significant improvement of the fit could only be obtained by introducing the energy of the $[70,1^-]_{N=1}$ as an additional free parameter (our fit (*d*)). The 1990 RPP also lists an ill determined 1*-state $N(2090)$ that summarizes all structures seen above 1800 MeV. In particular the KH-analysis reports a state at 1880 MeV, while the CMU-LBL-analysis finds a broad structure at 2180 MeV. Such states can clearly be accommodated by our model. With regard to states predicted at energies of 2300 MeV or above we must caution that these predictions depend strongly on the ill determined parameter Δ_2.

$\boldsymbol{D_{13}}$: Here similar observations are made as for the partial wave S_{11}. The 1990 RPP reports two resonances $N(1520)$ and $N(1700)$ with 4*-status and 3*-status respectively. Again, for both of them the wave function is predominantly contained in the $[70,1^-]_{N=1}$ multiplet, but with significant admixtures from the $[56,1^-]_{N=3}$ multiplet and from the high lying $[70,1^-]^b_{N=3}$ multiplet respectively. The theoretical values are too low by roughly 80 to 90 Mev. The same discussion applies to this discrepancy as in the case of the partial wave S_{11}. At higher energy the 1990 RPP reports evidence for a 2*-structure named $N(2080)$, which according to the CMU-LBL-analysis might really consist of two resonances at 1880 MeV and 2060 MeV, while the KH-analysis finds evidence only for a resonance at 2081 MeV. With wave functions

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[3] According to fit (*c*) the composition is

$$N(939) = 0.937\,[56(\tfrac{1}{2}),0^+]_{N=0} - 0.281\,[56(\tfrac{1}{2}),0^+]_{N=2} - 0.205\,[70(\tfrac{1}{2}),0^+]_{N=2} + \ldots,$$

$$N(1440) = 0.287\,[56(\tfrac{1}{2}),0^+]_{N=0} - 0.958\,[56(\tfrac{1}{2}),0^+]_{N=2} - 0.005\,[70(\tfrac{1}{2}),0^+]_{N=2} + \ldots.$$

[4] According to fit (*c*) the composition is

$$N(1720) = 0.875\,[56(\tfrac{1}{2}),2^+]_{N=2} - 0.447\,[70(\tfrac{1}{2}),2^+]_{N=2} + 0.187\,[70(\tfrac{3}{2}),0^+]_{N=2} + \ldots,$$

$$N(1680) = 0.901\,[56(\tfrac{1}{2}),2^+]_{N=2} - 0.433\,[70(\tfrac{1}{2}),2^+]_{N=2} + \ldots$$

[5] According to fit (*c*) the composition is

$$N(1535) = 0.892\,[70(\tfrac{1}{2}),1^-]_{N=1} + 0.230\,[70(\tfrac{3}{2}),1^-]_{N=1} - 0.347\,[56(\tfrac{1}{2}),1^-]_{N=3} + \ldots,$$

$$N(1650) = -0.143\,[70(\tfrac{1}{2}),1^-]_{N=1} + 0.938\,[70(\tfrac{3}{2}),1^-]_{N=1} + 0.225\,[56(\tfrac{1}{2}),1^-]_{N=3} + \ldots.$$

that come mainly from the $[56, 1^-]_{N=3}$ multiplet and the low lying $[70, 1^-]^a_{N=3}$ multiplet these states can easily be accommodated by our model. For higher energies the earlier remarks apply here likewise.

$\boldsymbol{D_{15}}$: Only one resonance $N(1675)$ with 4*-status is known in this partial wave. The wave function is an almost pure $[70(\frac{3}{2}), 1^-]_{N=1}$ state. Not surprisingly we find again that the mass calculated from the model is too low by about 90 MeV. At the high energy end the 1990 RPP lists a state $N(2200)$ with 2*-status and a not well defined mass. The model predicts for this energy two closely neighbouring states whose wave functions are superpositions from the $[56, 3^-]_{N=3}$ and the low lying $[70, 1^-]_{N=3}$ multiplets. As for higher energies our previous comment applies.

$\boldsymbol{G_{17}}$: There is one 4*-resonance $N(2190)$ in this partial wave, which fits into the model in a satisfactory manner. In fit (*c*) the wave function is predominantly $[56(\frac{1}{2}), 3^-]_{N=3}$ with some admixture of $[70(\frac{1}{2}), 3^-]_{N=3}$, in fit (*d*) the relation vice versa.

$\boldsymbol{G_{19}}$: This partial wave contains a well established 4*-resonance $N(2250)$. The wave function comes entirely from the $[70, 3^-]_{N=3}$ multiplet, and the model could easily fit the experimental mass value by simply adjusting the parameter Δ_2. One then obtains a quite reasonable value $\Delta_2 = -22$ MeV (our fit (*a*)). In contrast, the unsatisfactory value of our fit (*c*) stems from the fact that we have used Δ_2 as a completely free parameter and gave no special consideration to this particular resonance, but instead aimed at an optimal fit of all 4*-resonances.

Delta States with Positive Parity

$\boldsymbol{P_{31}}$: The existence of a 1*-resonance at 1550 MeV, which is seen only in inelastic channels, is classified as "doubtful" in the 1990 RPP. The Isgur-Karl model provides no space for a state with such a low mass. The two states predicted by the model are close together in mass and presumably can not be resolved by the experimental phase shift analysis, the KH and the CMU-LBL analyses both find only one state $\Delta(1910)$ with a large width of over 200 MeV, see however Chew [27].

$\boldsymbol{P_{33}}$: Our calculation has used the experimental $\Delta(1232)$ mass as an input with a very high weight, so that the mass given by the model must coincide with the experimental value. For the next state $\Delta(1600)$, a 2*-resonance, according to the 1990 RPP "the various analyses are not in good agreement", the masses range from 1522 MeV (KH) and 1600 MeV (CMU-LBL) up to 1690 MeV, all authors find a very large width 200–300 MeV. Our theoretical value of 1695 MeV is at the upper end of this range of uncertainty. The 3*-state $\Delta(1920)$ seems to be well represented by the model, comparing the theoretical value of 1932 MeV to the experimental values 1868 MeV (KH) and 1920 MeV (CMU-LBL), again the state has a width of 200–300 MeV. It is interesting to note that Chew [27] finds in this energy region two resonances with smaller widths and with masses of 1955 MeV and 2065 MeV respectively, as indeed the theoretical calculation seems to indicate.

$\boldsymbol{F_{35}}$: The $\Delta(1905)$ 4*-resonance is clearly seen in the KH and the CMU-LBL phase shift analyses. Another 2*-state $\Delta(2000)$ has been found only in the inelastic channel $N\pi \to N\pi\pi$, mainly $N\pi \to N\varrho$ [28]. Both states are well described by the Isgur-Karl model. Contrary to experiment, however, the theoretical prediction for the F_{35} state $\Delta(1905)$ is higher than that for the F_{37} state $\Delta(1950)$. This follows from the fact that both these states are predominantly $[56(\frac{3}{2}), 2^+]_{N=2}$ states, they are split solely by the hyperfine interaction.

$\boldsymbol{F_{37}}$: The 4*-resonance $\Delta(1950)$ appears in the KH-analysis as a well defined state at 1913 MeV, in agreement with our theoretical value. The second F_{37} state $\Delta(2390)$ reported in the 1990 RPP would require $N=4$ oscillator wave functions to be included in the calculation, which has not been done in this work.

Delta States with Negative Parity

$\boldsymbol{S_{31}}$: Experimental πN phase shift analyses find an energy difference of 70–90 MeV between the $\Delta(1620)$, which is the lowest S_{31} state (with 4*-rating), and the $\Delta(1700)$, which is the lowest D_{33} state (with 4*-rating). In contrast to this all our fits yield an energy difference of only 2–4 MeV. Both states come predominantly from the $[70, 1^-]_{N=1}$ multiplet with small admixtures from the high lying $[70, 1^-]_{N=3}$ multiplet. In the Isgur-Karl model they can, therefore, only be split by the hyperfine interaction, but this interaction is unable to provide a splitting of the required size of about 80 MeV. There is no way to resolve this problem within the nonrelativistic model, however Capstick and Isgur [29] have been able to overcome this difficulty in their "relativized" calculation. In our fits (*b*) and (*c*) the theoretical mass for the $\Delta(1900)$ 3*-resonance comes out too high by about 150 MeV. According to the model this state is an almost pure $[56, 1^-]_{N=3}$ state. Similar findings in the D_{33} and D_{35} partial waves suggest that the $[56, 1^-]_{N=3}$ multiplet as a whole is too high by about 130 MeV, presumably because the anharmonic terms of the confinement potential affect this multiplet in an unrealistic manner. In fact, Richard and Taxil [20] show that this multiplet is very sensitive to details of the anharmonic perturbation. In fit (*d*) the energy of the $[56, 1^-]_{N=3}$ multiplet was introduced as an additional parameter which led to a drastic improvement in the description of these states. There is some inconclusive (1*-status) experimental evidence for a third resonance $\Delta(2150)$ in the partial wave S_{31}, according to the model it would be a rather pure state from the low lying $[70, 1^-]_{N=3}$ multiplet.

$\boldsymbol{D_{33}}$: The problem connected with a simultaneous fit of the D_{33} resonance $\Delta(1700)$ and the S_{31} resonance $\Delta(1620)$ has been just discussed. The second state seen in the partial wave D_{33} is $\Delta(1940)$. Its status is rather uncertain (1*-rating), theoretically it should be a very pure $[56, 1^-]_{N=3}$ state and very likely, as discussed above, the theoretical mass is too large.

$\boldsymbol{D_{35}}$: Phase shift analyses agree on a state $\Delta(1930)$ with a 3*-rating. Theoretically this state would be a pure $[56, 1^-]_{N=3}$ state, in fit (*c*) the predicted mass is too high by about 130 MeV as a consequence of the fact that the whole $[56, 1^-]_{N=3}$ multiplet is too high by this amount. Only when the mass of this multiplet is treated as a free parameter can a good fit of the mass be obtained (fit (*d*)).

$\boldsymbol{G_{37}}$: Only one very broad resonance $\Delta(2000)$ with 1*-status is known. Phase shift analyses agree rather well on a mass value between 2200 and 2280 MeV, in agreement with the theoretical calculation. The assignment of this resonance to the $[56(\frac{3}{2}), 3^-]_{N=3}$ or the $[70(\frac{1}{2}), 3^-]_{N=3}$ multiplet depends strongly on the value chosen for Δ_2.

$\boldsymbol{G_{39}}$: Experimentally one extremely broad 2*-resonance $\Delta(2400)$ is known. The mass values given range from 2200 to 2468 MeV. Our model describes this state as pure $[56, 3^-]_{N=3}$ with a mass of 2236 MeV.

7. Relation to Other Theoretical Works

Closest related to our work is that of C. P. Forsyth and R. E. Cutkosky [11, 12]. While our own work has tried to remain as closely as possible within the original formulation of the Isgur-Karl model, these authors have introduced a number of modifications: Most importantly the contact term in the hyperfine interaction was allowed to assume different strengths for even parity and odd parity resonances, also the relative strength of the contact and tensor terms in the hyperfine interaction was taken as an adjustable parameter, moreover a certain amount of spin-orbit interaction was introduced with a strength different for even parity and odd parity resonances [6]. With the additional parameters at their disposal their fit of the resonance energies is better than our fit (*c*) (unmodified Isgur-Karl model). Our fit (*d*), where we introduced the energies of the $[70, 1^-]_{N=1}$ and $[56, 1^-]_{N=3}$ multiplets as additional free parameters, is the one more comparable to their calculation. In regard to configuration mixing we find rough agreement for the well established low lying excited states, but for energetically high excited states predictions are uncertain in some cases. For example, the G_{37} resonance $\Delta(2200)$ is assigned by our fit (*c*) to the $[56, 3^-]_{N=3}$ multiplet (in agreement with Corvi [10]), but by our fit (*d*) to the $[70, 3^-]_{N=3}$ multiplet (in agreement with Forsyth and Cutkosky). A similar observation holds for the G_{17} 4*-resonance $N(2190)$. Our fits (*c*) and (*d*) differ, aside from the two additional parameters in fit (*d*), in their very different values of the parameter Δ_2, which has a strong influence on the energy and composition of these resonances.

Other works where the $N = 3$ oscillator shell has been taken into consideration employ calculational methods different from that of the Isgur-Karl model. An extensive study by Böhm [30] used the variational method, where a specific form for the confinement

[6] It is not clear to us from their paper whether or not the mixing between the $N = 1$ and the $N = 3$ oscillator shells arising from the anharmonic confining potential U was taken into account.

potential is assumed, which is then approximately diagonalized by using harmonic oscillator wave functions as test functions, and by optimizing the harmonic oscillator frequencies ω_ϱ and ω_λ independently for each SU_3 multiplet and each value of orbital angular momentum. The oscillator frequencies so obtained vary from state to state and over a considerable range, in contrast to the fixed values for ω_ϱ and ω_λ in the Isgur-Karl approach. Configuration mixing due to the confinement potential is minimal in this method. More recently C. S. Kalman and B. Tran have used the variational approach to study the whole baryon sector including all flavors [31].

Many authors have investigated various properties of the non-relativistic constituent quark model. In a study restricted to the spin independent part of the 3-quark hamiltonian B. Silvestre-Brac and C. Gignoux have compared calculations using a truncated harmonic oscillator basis (as in the Isgur-Karl model) with results deriving from a solution of the exact 3-body Faddeev equations, they found very good agreement [32]. The non-relativistic kinematics of the model is an obvious deficiency which must be improved, at least by introducing relativistic correction terms. Capstick and Isgur [29] have carried out a very detailed investigation in the baryon sector, surprisingly they find only little change in the gross structure of the resonance energy spectrum. The difficulty in fitting with one parameter set simultaneously the positive parity and negative parity states, which we have mentioned in the previous sections, also exists in the "relativized" calculation, Capstick and Isgur too feel compelled to shift the $N=1$ negative parity states upward relative to the positive parity states by about 90 MeV. The quasi-relativistic kinematics lead to a deeper understanding of why the spin-orbit interaction plays a comparatively unimportant role. Looking at other details, the relativistic smearing of the contact interaction removes, in accordance with experiment, some degeneracies that could not be removed in the non-relativistic calculation (e.g. the degeneracy between the S_{31} state $\Delta(1620)$ and the D_{33} state $\Delta(1700)$).

To improve on the energy of the $[70,1^-]_{N=1}$ multiplet relative to the positive parity states a number of remedies have been suggested. H. J. Weber and H. T. Williams propose to include a one-pion-exchange interaction between the constituent quarks [33]. Bhaduri and collaborators consider an anisotropic confinement potential leading to deformed baryon states [34]. Sharma et al. [35] investigate by the variational method the orbit-orbit component of the Fermi-Breit interaction and find that it can be used instead of the anharmonic part U of the confinement potential, and that the positive parity states are lowered relative to the negative parity states.

8. Concluding Remarks

In this paper we have extended the Isgur-Karl model to include the full set of $N=3$ oscillator states in order to be able to describe a larger set of resonance states in the negative parity sector. Because of the inclusion of the $N=3$ states one has additional configuration mixing due to the anharmonic part of the confinement potential between the $N=1$ and the $N=3$ oscillator shells. We have tried to fit positive parity and negative parity states simultaneously with the same set of parameters. Existing difficulties have been discussed on the basis of a detailed comparison with the resonance parameters deduced from the major pion-nucleon phase shift analyses. Various proposals for the improvement of constituent quark models have been reviewed. It appears to us that the Isgur-Karl model remains an interesting and useful starting point for improved calculations. It will continue to give qualitative new insight into the physics of baryons, although we do not expect its development into a truely quantitative model.

Not only the Isgur-Karl model, but likewise all other constituent quark models that attempt to determine baryon resonance energies by means of a bound state calculation, with neglect of all couplings to the decay channels, are subject to serious objections. These have been most clearly formulated by Höhler (see [15, 36] and his article in the 1990 Review of Particle Properties, p. VIII.9 in [21]). On the one hand, by turning on the couplings to decay channels, bound state poles are displaced into the complex energy plane, and theory really should try to predict these complex pole positions. On the other hand, there is no unique way to extract resonance parameters from the experimental data, and "experimental" resonance positions do not simply coincide with the real parts of the complex poles. Constituent quark models of the type of the Isgur-Karl model avoid the complexity of a many-channel calculation at the expense of having to identify resonance energies with the pole positions in a bound state calculation. Therefore, at best a semi-quantitative description of the baryon excited states can be expected. First attempts at dealing with the many-channel problem and calculating the complex pole positions have been made in the work by Blask, Huber, and Metsch [37], using a variational approach (see also [38]).

Acknowledgements

One of the authors (M.K.) expresses his gratitude for their kind hospitality to the members of the Physics Department of Florida State University at Tallahassee, where the final version of this paper was written.

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